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Total No. of Questions : 08

B.Tech (ECE) (Sem.–5)

DIGITAL SIGNAL PROCESSING

Subject Code : BTEC-502-18

M.Code : 78298

Date of Examination : 07-01-22

Time : 2 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **Attempt any FIVE question(s), each question carries 12 marks.**

1. What are similarities and dissimilarities between Circular and Linear Convolutions? Determine linear and circular convolutions the sequences $x[n]$=[2 1 2 1] and $h[n]$=[1 2 3]. Do state necessary modifications in signal sequences, wherever required.

2. Tabulate differences between Discrete Fourier Transform and Fast Fourier Transform. Explain algorithms that do are used to determine FFT with the help of an example.

3. Compare Chebyshev, Butterworth and Elliptic filters. Discuss steps involved in design of IIR Butterworth low pass filters using Bilinear Transformation method.

4. What is multirate signal processing? Discuss requirement of multirate sampling with examples?

5. Draw a fully labelled basic architecture of TMS series of Digital Signal Processors. Discuss various building blocks in brief.

6. What are FIR & IIR Filters? Explain their realization structures.

7. Assume $y[n]$ be the output sequence for an input sequence x[n]. Give suitable examples (mathematical representations) of systems that can be named as: (a) Memoryless (b) Time-Invariant (c) Causal (d) Stable (e) Linear Time Invariant

8. Relate Fourier Series (FS), Fourier Transform (FT), Discrete-Time Fourier Transform (DTFT), Discrete Fourier Transform (DFT) and Fast Fourier Transform (FFT); mathematically and conceptually.

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Ques2 :→ In the field of Digital Signal Processing (DSP), Fourier analysis is used to decompose the signals. The mathematical tool Discrete Fourier transform (DFT) is used to digitize the signals. The collection of various fast DFT computation techniques are known as the fast fourier transform (FFT). In simpler words, FFT is just an implementation of the DFT.

Difference :-

DFT	FFT
1. The DFT stands for Discrete Fourier transform.	1. The FFT stands for Fast Fourier transform.
2. The DFT is only applicable for discrete and finite-length signals. Discrete time-domain signals are transformed into discrete frequency domain signals using DFT.	2. It is an implementation of DFT.
3. $X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi k n}{N}}$	3. FFT mainly works with computational algorithms for the fast execution of DFT.
4. The time complexity required for a DFT to perform is equal to the order of N^2 or $O(N^2)$.	4. The time complexity reduces in the case of FFT and becomes equal to $O(N \log N)$.
5. The DFT has less speed than the FFT.	5. It is the faster version of DFT.
5. Applications :- spectral analysis, solution of partial differential equations, correlation analysis etc.	6. Applications :- Filtering algorithms, multiplication of integer and polynomials, etc.

DFT :→ The signal found in nature are basically analog type of signals. But the digital computers that are used for the analysis of the signals can work only with the information that is discrete in nature and finite in length. Hence, the digitization of signal is performed. The Fourier transform of a signal within a finite range is called Discrete Fourier transform.

The Discrete Fourier transform of a signal $x(n)$ is mathematically expressed as :-

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}}$$

where, $k = 0, 1, 2, \cdots N-1$

The signals are periodic in nature. The frequency response of a signal from its impulse response can be obtained from the DFT of the required signal.

- FFT :→ The fast fourier Transform (FFT) is nothing but an implementation of a very common tool of DSP called the DFT. The FFT provides a more efficient result than DFT. The computational time required for a signal in the case of FFT is much lesser than that of DFT. Hence, it is called Fast Fourier Transform which is a collection of various fast DFT computation techniques. The FFT works with some algorithms that are used for computation.

Conclusion :- The transformation of time-domain signals to frequency domain signals are the key part of DSP. As the time passes, the evolution of processes takes place. The FFT is updated version or way of implementation of the DFT that takes less computational time and more efficient results than that of ordinary DFTs.

* FFT Algorithm :- If we take the 2- point DFT and 4-point

DFT and generalize them to 8-point, 16-point, --- 2^r point, we get the FFT algorithm.

There are many variants of the FFT algorithm. There is decimation-in-time FFT algorithm for sequences whose length is a power of two ($N = 2^r$ for some integer r).

Butterflies and Bit reversal:- The FFT algorithm decomposes the DFT into $\log_2 N$ stages, each of which consists of $N/2$ butterfly computations. Each butterfly takes two complex no.'s p & q computes from them two other no.'s $p+\alpha q$ & $p-\alpha q$, where α is a complex no.'s.

(I) Decimation in time:- Let $x(n)$ be an N-point sequence, $0 \le n \le N-1$ and $N = 2^m$

In this algorithm, sequence $x(n)$ is decimated into two $\frac{N}{2}$ point sequence.

Even-indexed sequence:- $g(n) = \{x(0), x(2) \text{------} x(N-2)\}$
$= x(2n), \; 0 \le n \le \frac{N}{2}-1$

Odd-indexed sequence: $h(n) = \{x(1), x(3), \text{----} x(N-1)\}$
$= x(2n+1), \; 0 \le n \le \frac{N}{2}-1$

Ex:- Let $N = 8$,

$g(n) = \{x(0), x(2), x(4), x(6)\}$

$h(n) = \{x(1), x(3), x(5), x(7)\}$

Let $x(n) \xleftrightarrow[N]{DFT} X(k)$, $0, k \le N-1$

$g(n) \xleftrightarrow[N/2]{DFT} G(K)$, $0 \le K \le \frac{N}{2}-1$

$h(n) \xleftrightarrow[N/2]{DFT} H(K)$, $0 \le K \le \frac{N}{2}-1$

We know, $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$, $0 \le k \le N-1$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2(n)K} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_N^{(2n+1)K}$$

$$X(K) = G(K) + W_N^K H(K), \quad 0 \le K \le N-1 \quad \text{——— (1)}$$

$$\text{Also, } G(K) = G\left(K+\frac{N}{2}\right)$$

$$H(K) = H\left(K+\frac{N}{2}\right)$$

$$W_N^{K+\frac{N}{2}} = -W_N^K$$

$$\text{So, } X\left(K+\frac{N}{2}\right) = G\left(K+\frac{N}{2}\right) + W_N^{K+\frac{N}{2}} H\left(K+\frac{N}{2}\right)$$

$$\therefore \quad X\left(K+\frac{N}{2}\right) = G(K) - W_N^K \, \& \, H(K), \quad 0 \le K \le \frac{N}{2}-1 \quad \text{——— (2)}$$

Substiture value of K in eqn. (1)

$K=0, \; X(0) = G(0) + W_N^0 H(0)$

$K=1, \; X(1) = G(1) + W_N^1 H(1)$

$K=2, \; X(2) = G(2) + W_N^2 H(2)$

$K=3, \; X(3) = G(3) + W_N^3 H(3)$

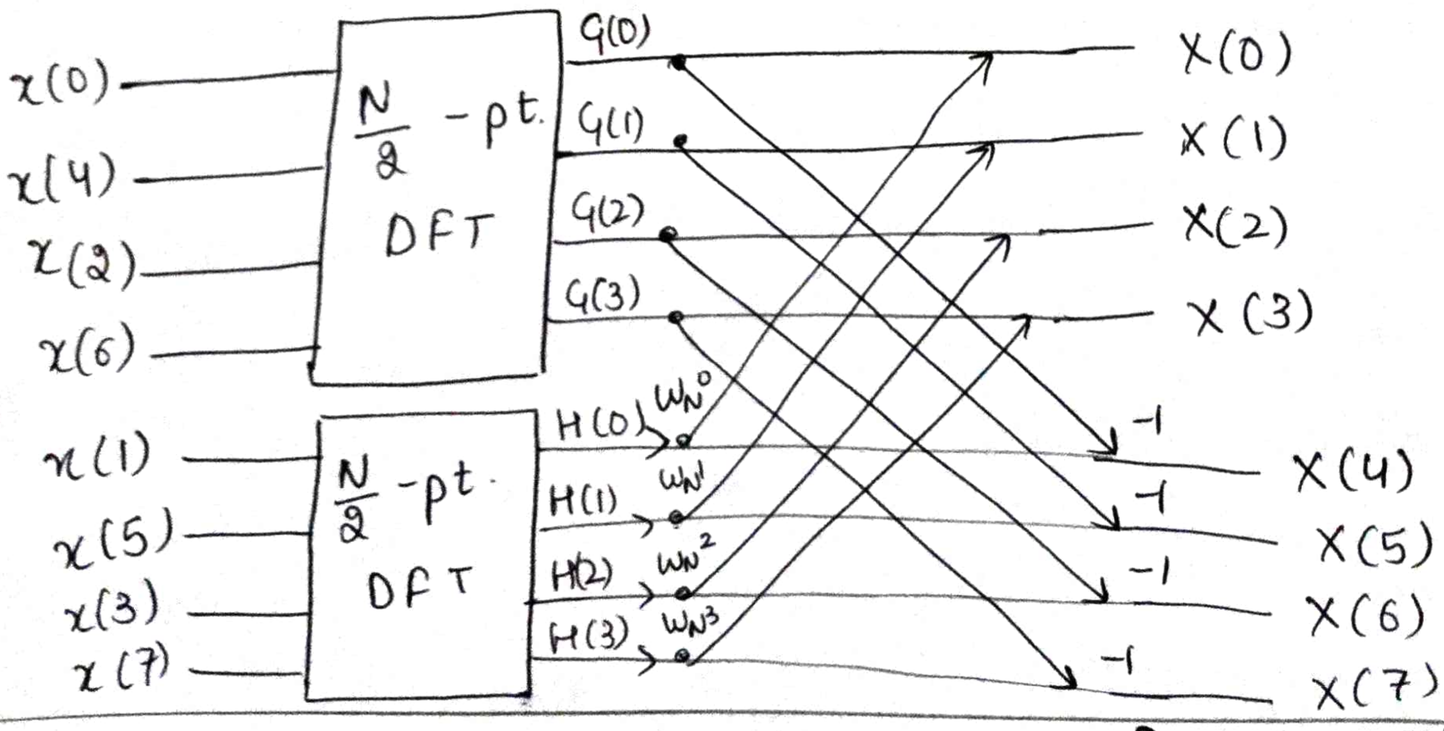
Now, in eqn. (2)

$K=0, \; X\left(0+\frac{8}{2}\right) = X(4) = G(0) - W_N^0 H(0)$

$K=1, \; X(5) = G(1) - W_N^1 \, \& \, H(1)$

$K=2, \; X(6) = G(2) - W_N^2 H(2)$

$K=3, \; X(7) = G(3) - W_N^3 H(3)$



Ques 3: → As we know filter is the module which passes certain frequencies and stops certain frequencies as designed. Based on technical design specifications there are filter types such as Butterworth, Chebyshev, Elliptic filter.

① Butterworth filter :- Following are the major unique characteristics of the butterworth filter.

- It has maximally flat respone within the passband of the filter.
- It has moderate phase distortion.

Butterworth LPF will have all the poles and they will be located on the unit circle with equal angles.

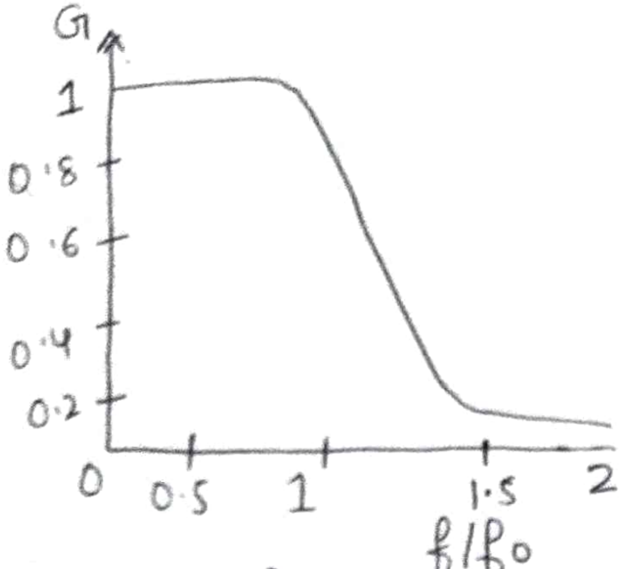
② Chebyshev Filter:- It allow ripples in the passband amplitude response. It is also known as ripple response filter. The amount of ripple is provided as one of the design parameter for this type of chebyshev filter.

This filter will have steeper roll-off near cut off frequency in comparison to butterworth filter. But this results into monotonicity in passband region along with poor transient response.

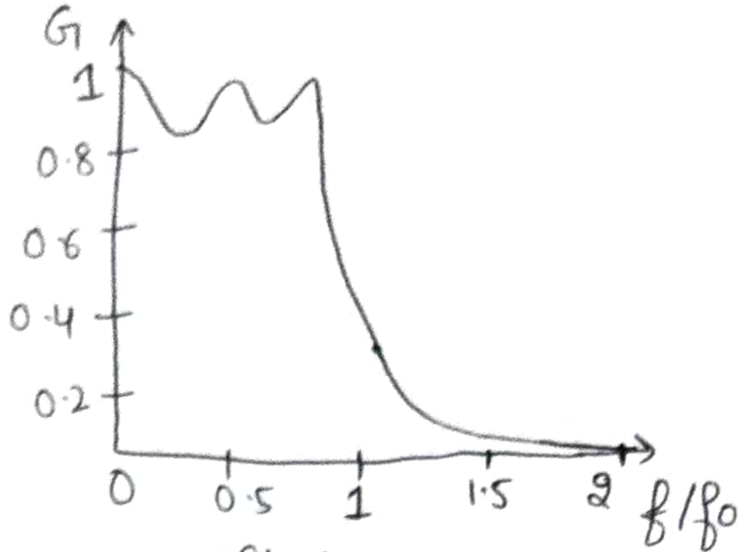
Characteristics :-
- Ripple in passband.
- Sharper transition band
- Poorer group delay.

3) Elliptic filter :- • In this type of elliptic filter cutoff slope is sharper compare to all other filters.

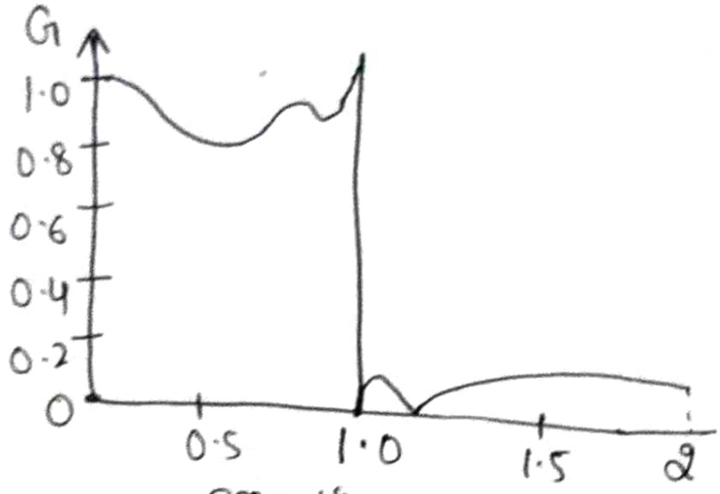
- But it will have ripples in passband and stopband of amplitude response.
- It will have very non-linear phase response.



Butterworth



Chebyshev



Elliptic

- Design of IIR Butterworth low pass filter using BTM :→

First step is to convert analog specifications to digital specifications : $\frac{F_s}{2\pi} = \frac{f_p}{\omega_p}$, hence $\omega_p = 2\pi \frac{f_p}{F_s}$

and $\omega_s = 2\pi \frac{f_s}{F_s}$

Converting the criteria relative to the digital normalized scale gives

$$20 \log |H(e^{j\omega_p})| \geq \delta_p$$

$$20 \log |H(e^{j\omega_s})| \leq \delta_s$$

Hence

$$|H(e^{j\omega_P})| \geq 10^{\frac{\delta_P}{20}} \quad \text{—— (A)}$$

$$|H(e^{j\omega_S})| \leq 10^{\frac{\delta_S}{20}} \quad \text{—— (B)}$$

Butterworth analog filter squared magnitude fourier transform is given by,

$$|H_a(j\Omega)|^2 = \frac{1}{1+\left(\frac{j\Omega}{j\Omega_c}\right)^{2N}}$$

Now, A & B in terms of analog Butterworth frequency response & become:-

$$\frac{1}{1+\left(\frac{\Omega_P}{\Omega_c}\right)^{2N}} \geq \left(10^{\frac{\delta_P}{20}}\right)^2 = 10^{\frac{\delta_P}{10}}$$

$$\frac{1}{1+\left(\frac{\Omega_S}{\Omega_c}\right)^{2N}} \leq \left(10^{\frac{\delta_S}{20}}\right)^2 = 10^{\frac{\delta_S}{10}}$$

Now, $\Omega_P = \frac{2}{T}\tan\left(\frac{\omega_P}{2}\right)$, $\Omega_S = \frac{2}{T}\tan\left(\frac{\omega_S}{2}\right)$

$$\therefore \; 1+\left(\frac{\frac{2}{T}\tan\left(\frac{\omega_P}{2}\right)}{\Omega_c}\right)^{2N} \leq 10^{\frac{\delta_P}{10}} \text{ —①}; \; 1+\left(\frac{\frac{2}{T}\tan\left(\frac{\omega_S}{2}\right)}{\Omega_c}\right)^{2N} \geq 10^{\left(\frac{\delta_S}{10}\right)} \text{ —②}$$

Changing inequalities to equalities,

$$\left(\frac{\frac{2}{T}\tan\left(\frac{\omega_P}{2}\right)}{\Omega_c}\right)^{2N} = 10^{\frac{\delta_P}{10}}-1 \; ; \; \left(\frac{\frac{2}{T}\tan\left(\frac{\omega_S}{2}\right)}{\Omega_c}\right)^{2N} = 10^{\frac{\delta_S}{10}}-1$$

Diving the above ② results in;

$$N = \frac{1}{2}\left[\frac{\log\left(10^{\frac{\delta_P}{10}}-1\right) - \log\left(10^{\frac{\delta_S}{10}}-1\right)}{\log\left(\tan\left(\frac{\omega_P}{2}\right)\right) - \log\left(\tan\left(\frac{\omega_S}{2}\right)\right)}\right]$$

For bilinear Transformation:-

$$\left[\frac{1+\frac{2}{T}\tan\left(\frac{w_s}{2}\right)}{\Omega_c}\right]^{2N} = 10^{\delta_{s}/10}$$

By solving, $\Omega_c = \dfrac{\frac{2}{T}\tan\left(\frac{w_s}{2}\right)}{10\left(\frac{1}{2N}\log_{10}\left(10^{\frac{\delta_s}{10}}-1\right)\right)}$

Now, poles of H(s); $|H_a(s)|^2 = \dfrac{1}{1+\left(\frac{s}{j\Omega_c}\right)^{2N}}$

For bilinear H(s) is given by;

$$H_a(s) = \frac{K}{\prod_{i=0}^{N-1}(s-s_i)} \quad -③$$

$$s_i = \Omega_c e^{j\left(\frac{\pi(1+2i+N)}{2N}\right)} = \Omega_c\left(\cos\frac{\pi(1+2i+N)}{2N} + j\sin\frac{\pi(1+2i+N)}{2N}\right]$$

Then,

$$H_a(s) = \frac{K}{\prod_{i=0}^{N-1}\left(s-\Omega_c\left(\cos\frac{\pi(1+2i+N)}{2N} + j\sin\left(\frac{\pi(1+2i+N)}{2N}\right)\right)\right]} \quad -④$$

where

$$\Omega_c = \frac{\frac{2}{T}\tan\left(\frac{w_s}{2}\right)}{10\left(\frac{1}{2N}\log_{10}\left[10^{\frac{\delta_s}{10}}-1\right)\right)}$$

Ques 4: → Multirate signal processing:- The implementation of a digital signal processing application using variable sampling rates.

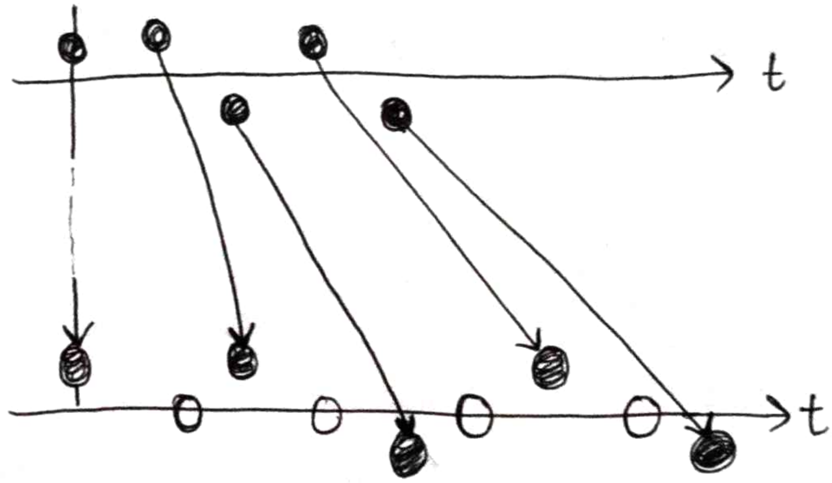
- Upsampling increases the sample rate.
- Downsampling reduces the sampling rate.

~~Oper~~

Operations of Multirate Signal Processing :-

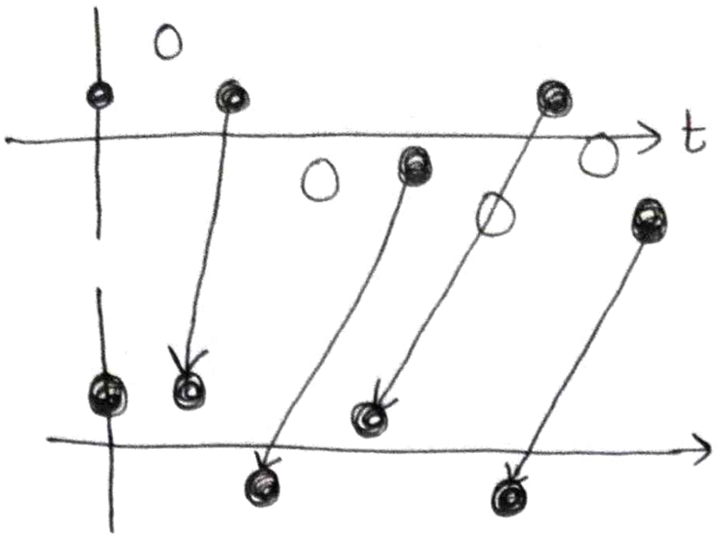
① Upsampling or Interpolation :-

- For an unsampling by a factor of I, add I-1 zeroes b/w samples in the original sequence.
- An upsampling by a factor I is commonly written ↑I.
  For example, upsampling by two : ↑ 2
- Obiviously the no. of samples will approx. double after ↑2.
- Note that if the sampling frequency doubles after an upsampling by two, that the original sample sequence will occur at the same points in time.



② Downsampling or Decimation :-

- To decimate by a factor D, keep one of every D samples - on a periodic basis.
- Downsampling by a factor I is commonly written ↓I For e.g :- downsampling by two : ↓2.
- Obiviously the no. of samples will be approx. cut in half after ↓2.



Requirements :- ① Dual Tone Multifrequency Signal Detection :→ Dual tone multifrequency signaling, increasingly being employed worldwide with push-button telephone sets offers high dialing speed over the dial pulse signaling.

② Can improve the flexibility of a software.

③ Reduces the need for expensive anti-aliasing analog filters.

④ Enables processing of different types of signals with different sampling rates.

⑤ Can lead to a significant saving in computational power.

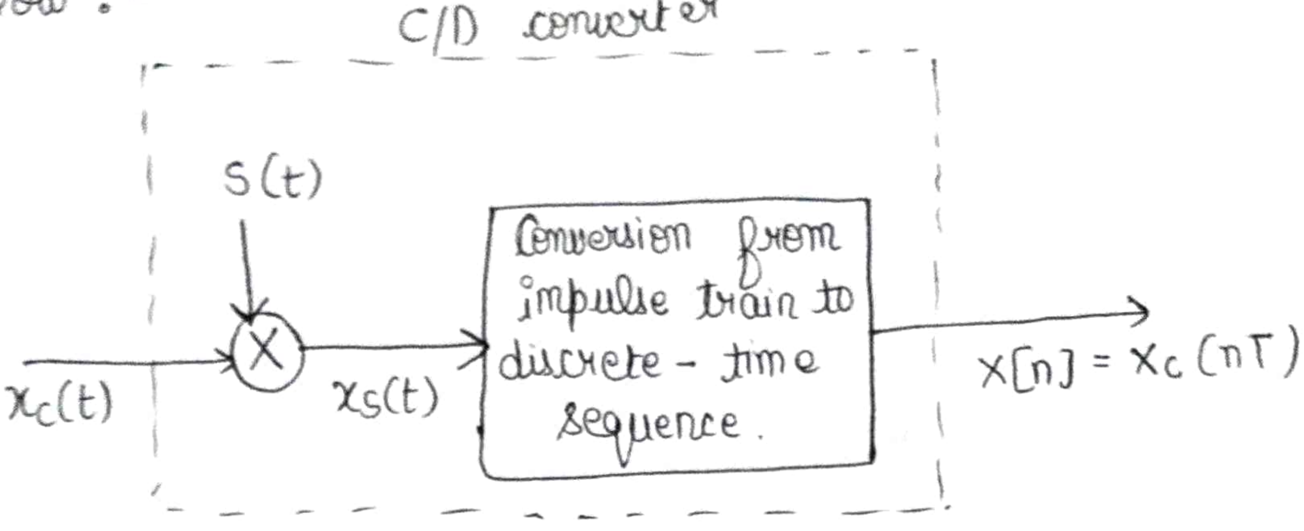
⑥ Wideband receivers take advantage of Multirate signal processing for efficient channelization.

⑦ Offers flexibility for symbol synchronization and down-conversion of software radios.

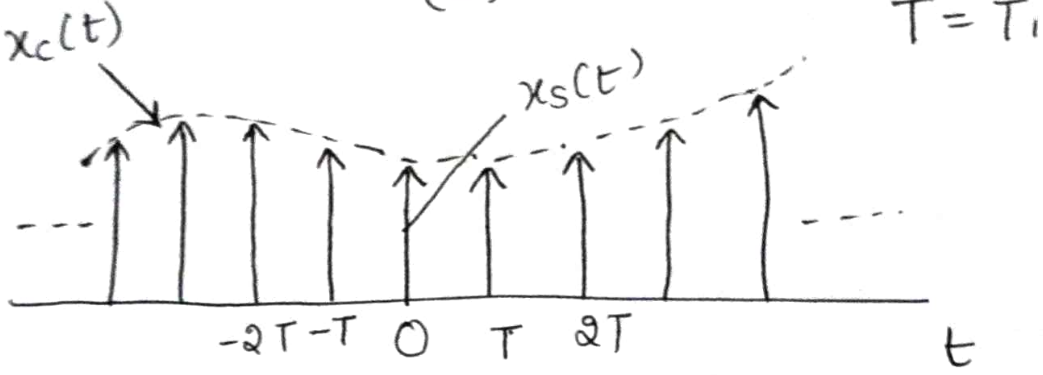
⑧ It allows partioning of the high-speed processing into parallel multiple lower speed processing tasks which reduces the cost.

⑨ It is widely required in compact disc, digital audio, TV's etc.

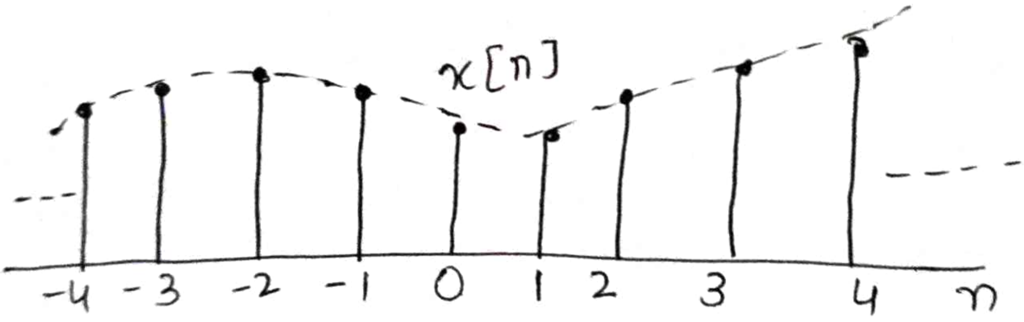
Example :- Multirate DSP helps in A/D conversion as shown below :-



(a)



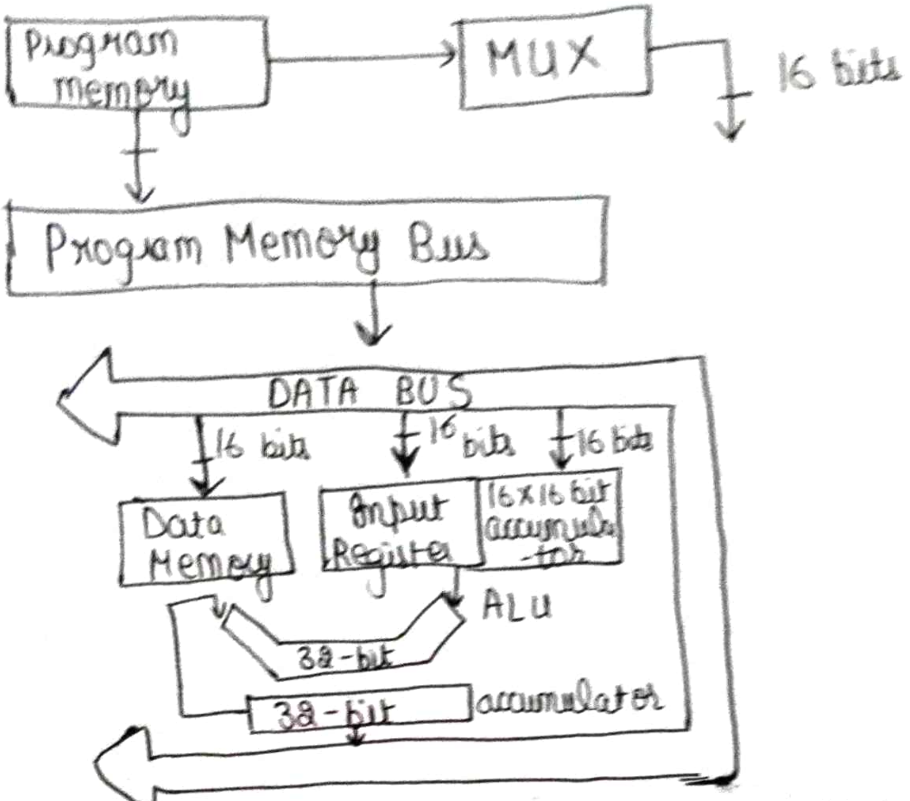
↓



In multirate digital signal processing the sampling rate of a signal is changed in order to increase the efficiency of various signal processing operations. Decimation or down sampling reduces the sampling rate, whereas expansion or up-sampling, fol-lowed by interpolation increases the sampling rate.

Ques 5:→ The TMS 320 Cxx series of DSP processor is manufactured by Texas instruments.

① TMS 320 C_{10} (1st generation)



1st generation DSP processor was introduced in 1982.

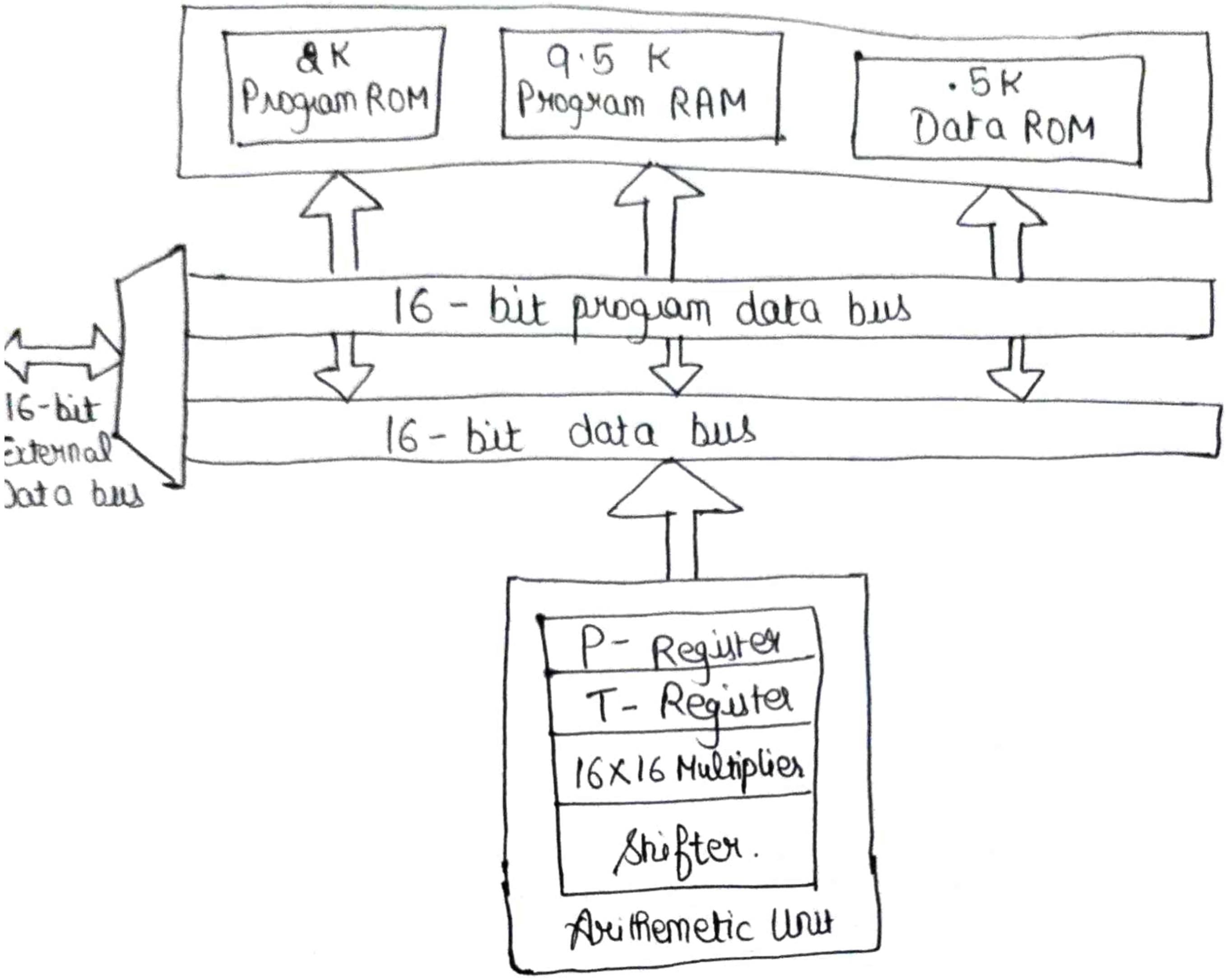
Key feature include :-

Arithmetic unit :- which includes multiplexer, accumulator, architecture and shifter. The processor family has modified, Harward architecture with two separate memory space for program and data. It has an chip memory & special instructions for execution of basic DSP algorithm.

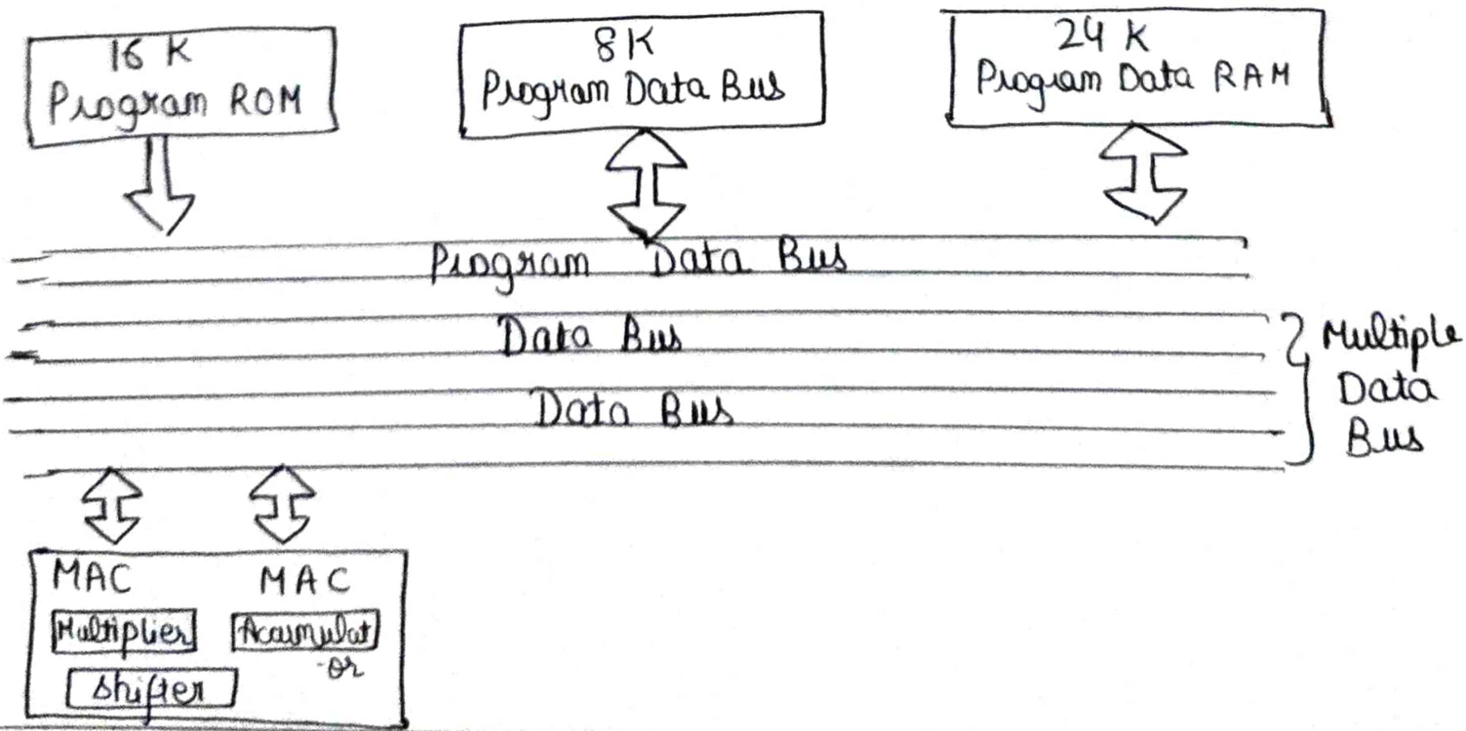
② TMS 320 C_{50}C (IInd generation)

- IInd generation fixed point DSP processor have enhanced features compared to 1st generation.
- These include much larger chip memory and more special instruction to support the efficient execution of DSP algorithm.

- As a result computational performance of IInd generation DSP processor is 4 or 6 times than that of 1st generation.

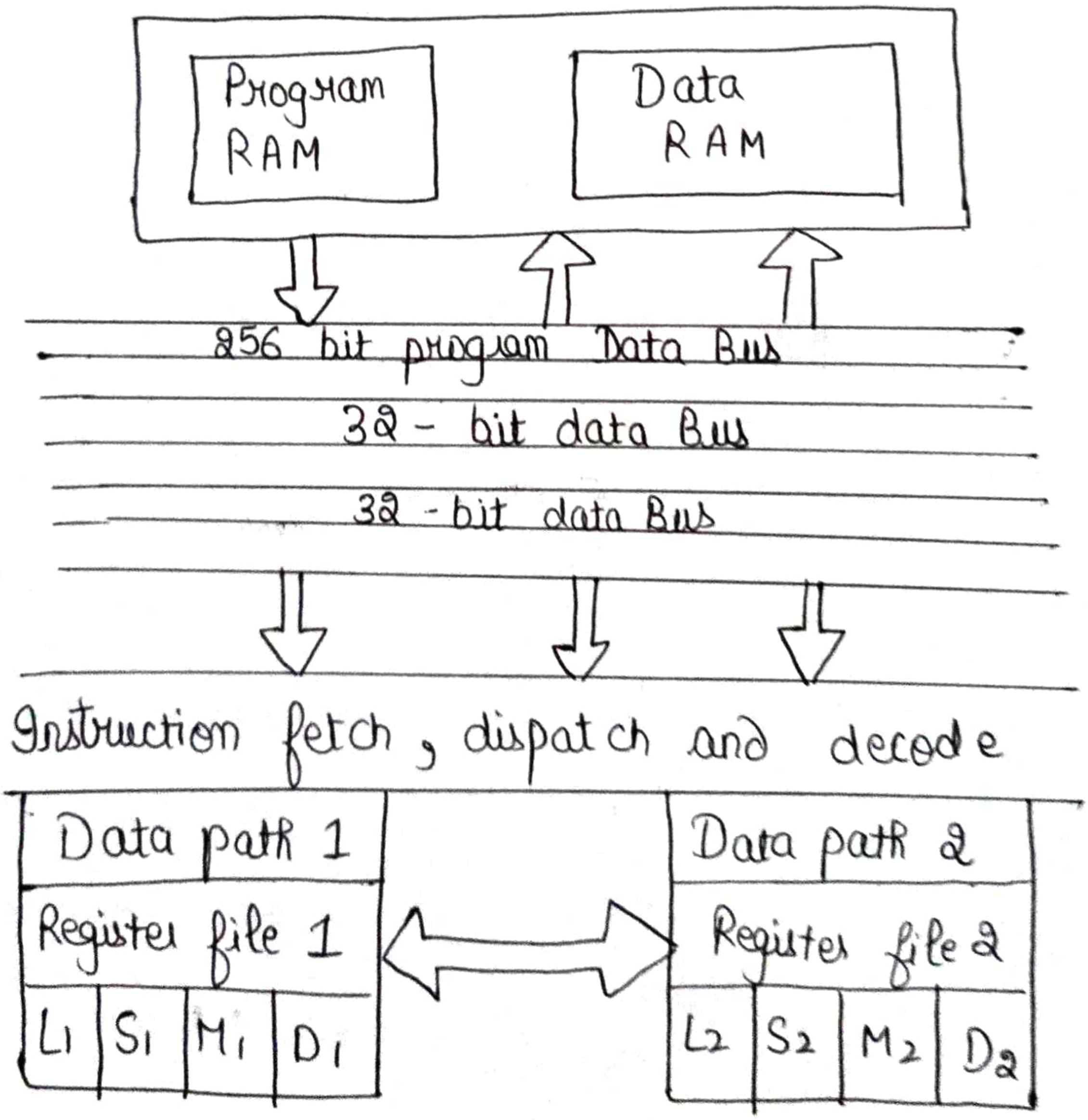


③ TMS 320 C54 (IIIrd Generation)



- In this the performance enhancement is increased or making more effective use of available on chip resources.
- It includes more and wider data parts.
- The performance of IIIrd generation processor is 2 or 3 times superior to that of IInd generation.
- They are mainly used in digital communication and digital audio.
- MAC includes multiplier, exponent, detector, comparison and negation.

④ TMS 320 C62 (IV generation)



- It aimed at large emerging multichannel applications such that remote access, sever mode, wireless base station and medical imaging, digital subscribers loop etc.
- It includes two independent arithmetic parts each with 4 execution units i.e., logic unit, shifter, multiplexer and data address unit.
- It has a large program and data cache memory, each part has its own register file but it can also access register on other data parts.

Ques 6 :→ FIR filters :- A finite impulse response (FIR) filter is a filter whose impulse response is of finite duration. because it settles to zero in finite time. FIR filter can be discrete-time or continuous time, and digital or analog.

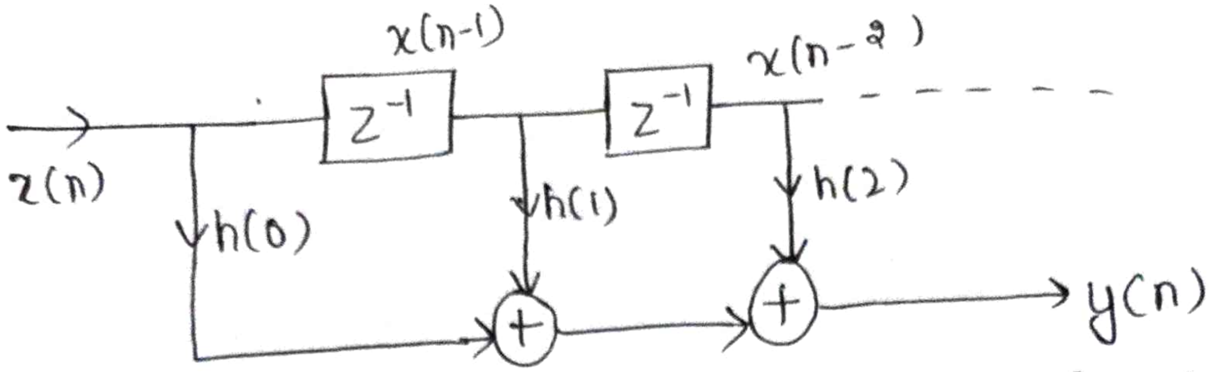
Structure of FIR :-

① Direct form :-

$$y(n) = \sum_{k=0}^{M-1} h(k) \cdot x(n-k)$$

$$\frac{Y(z)}{X(z)} = \sum_{n=0}^{M-1} h(n) z^{-n}$$

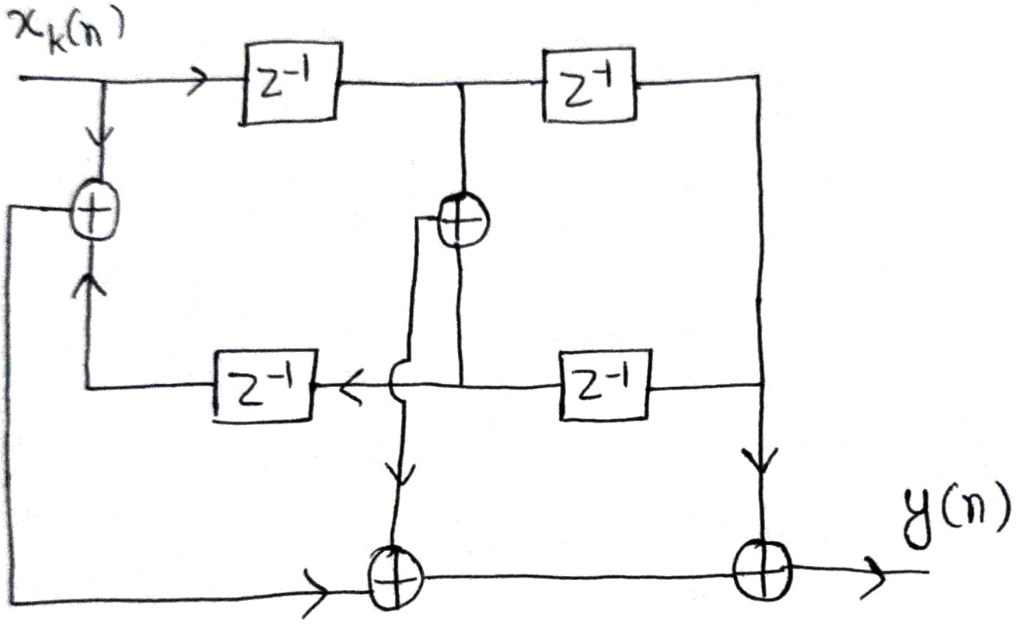
$$= h(0) + h(1) z^{-1} + h(2) z^{-2} + \cdots h(M-1) z^{-(M-1)}$$



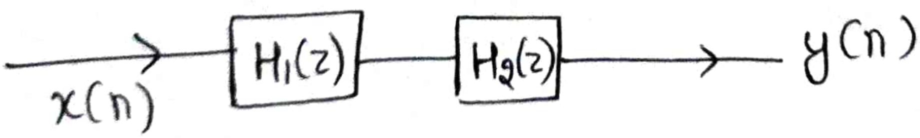
$$y(n) = h(0)\,x(n) + h(1)\,x(n-1) + h(2)\cdot x(n-2).$$

② **Cascade form** :→

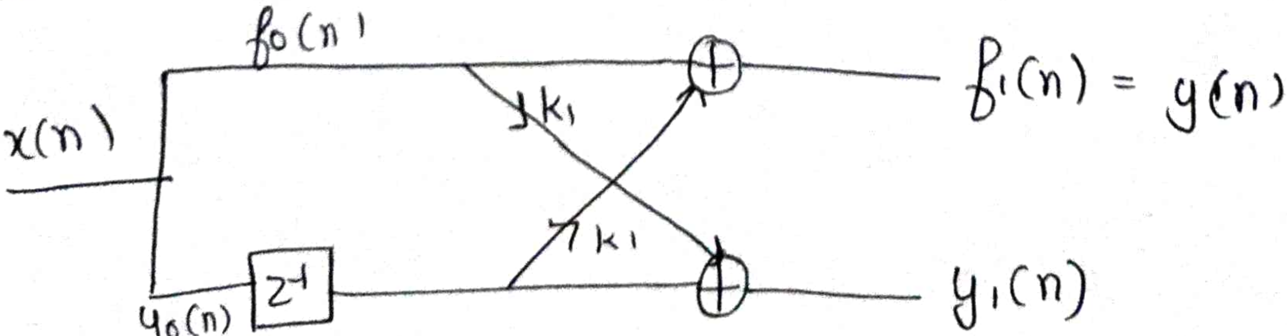
$$H(z) = H_1(z)\cdot H_2(z)$$



or

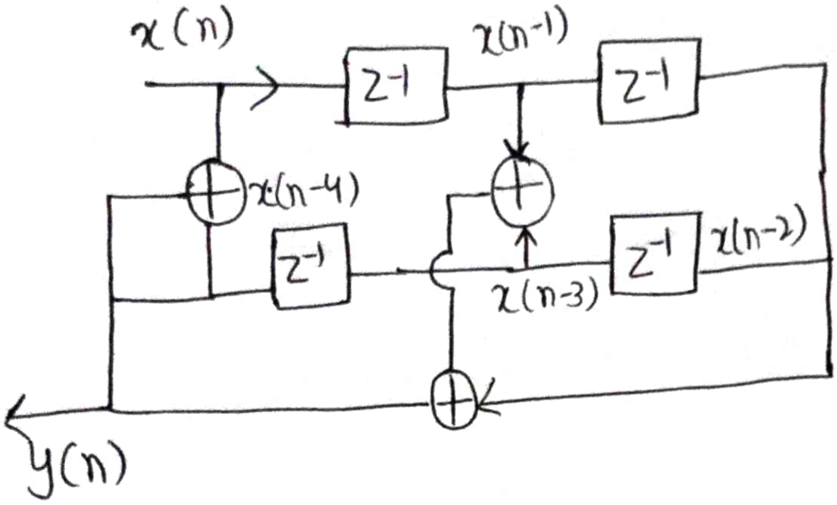


③ **Lattice form** :-



(4) Linear Phase Realization:

$$H(z) = \sum_{n=0}^{M-1} h(n) z^{-n}$$



IIR filters :- The infinite impulse response filters is a recursive filter in that o/p from the filter is computed by using the current & previous inputs and previous outputs, there is feedback for the output in the filter structure.

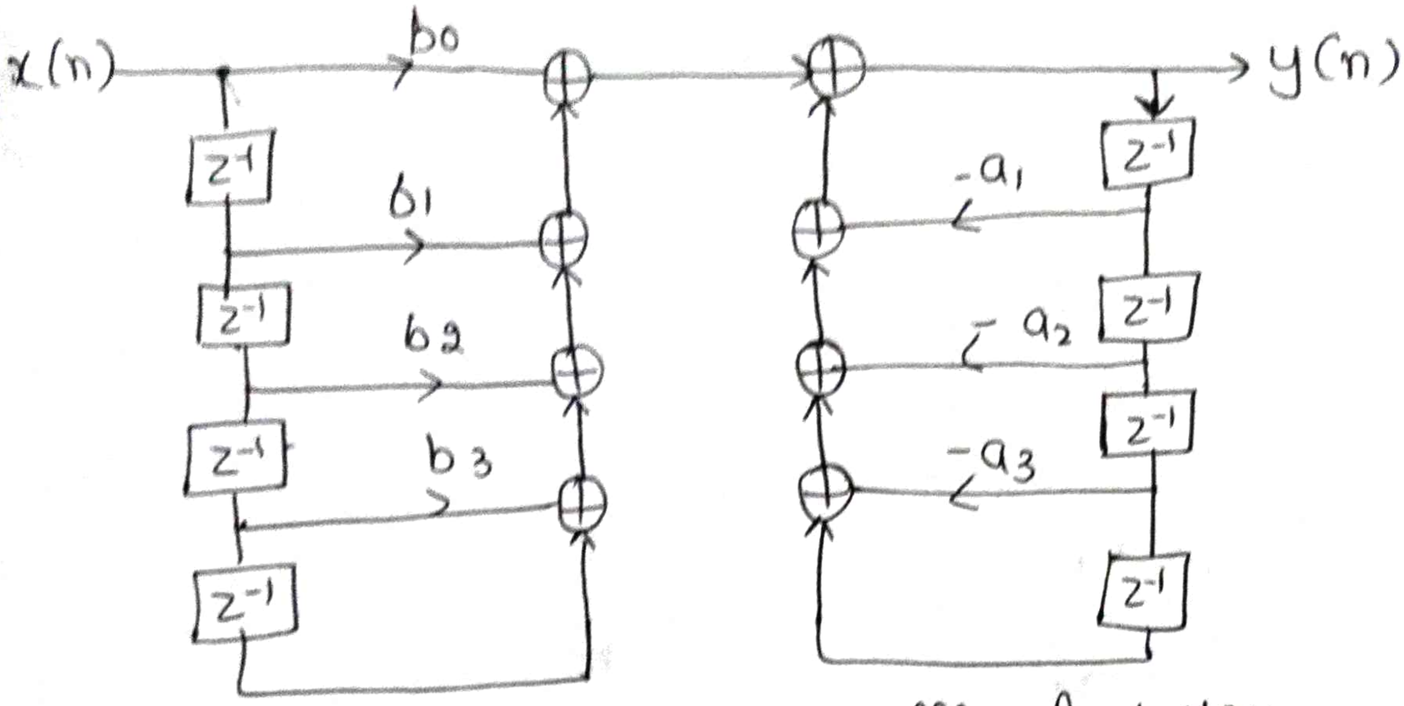
$$y(n) = \sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

Realization :→

(1) ~~I - Direct form (I)~~

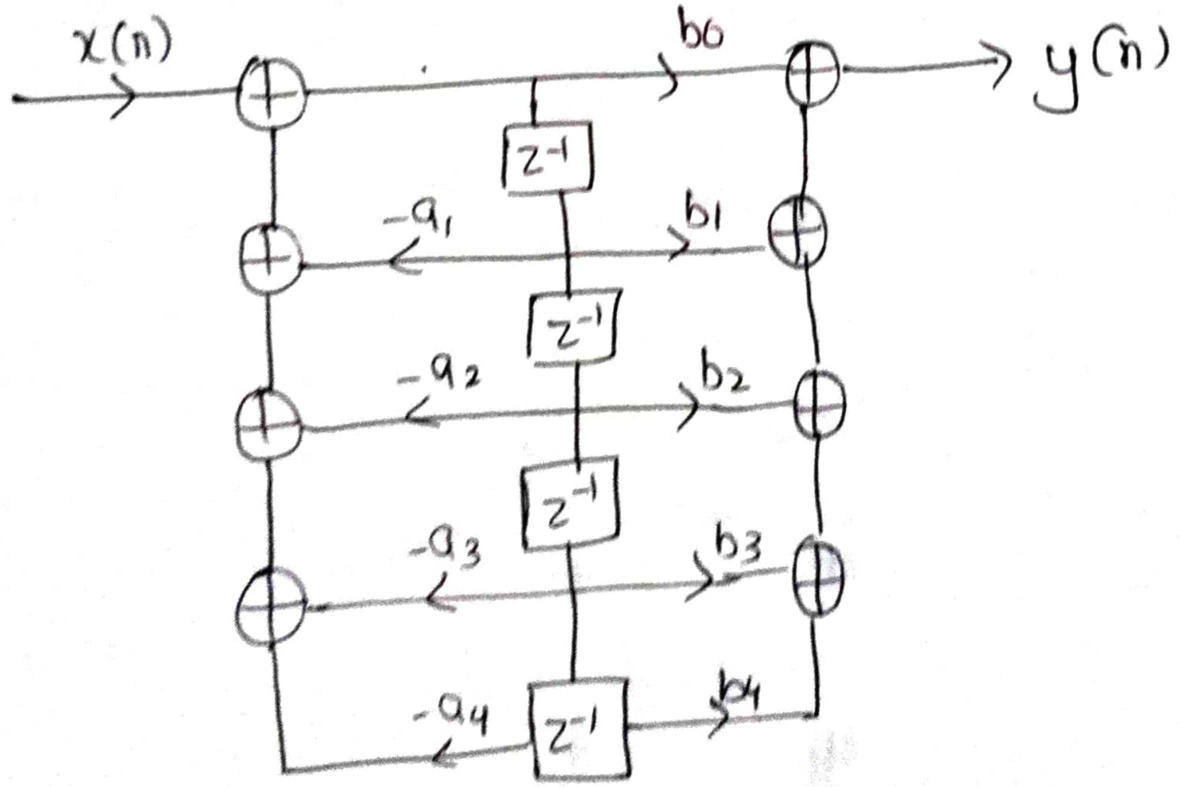
① Direct Form (I)



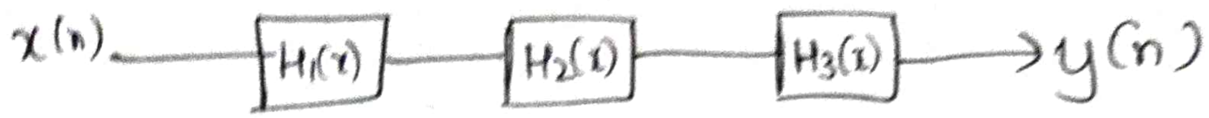
All zero system

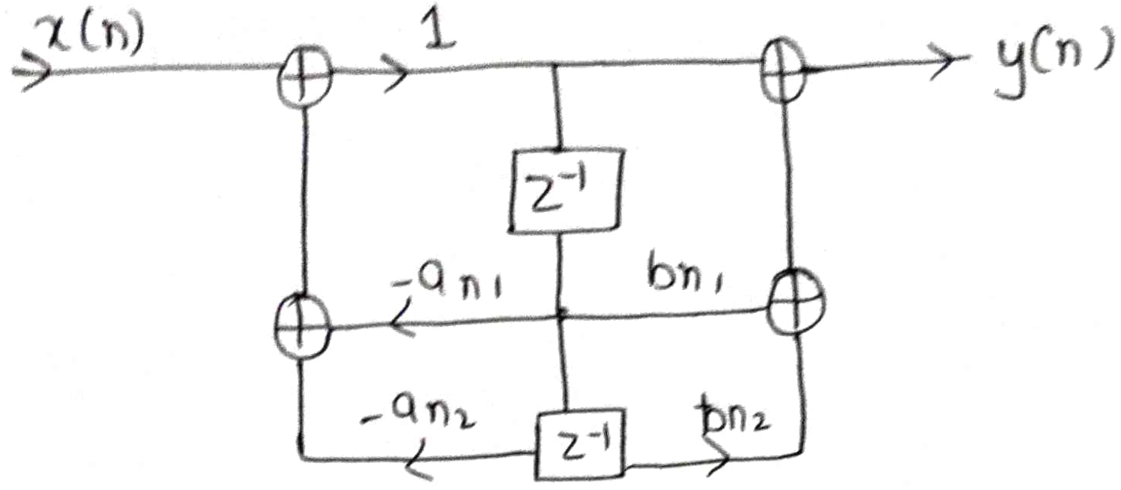
All pole system

Direct - form (II)

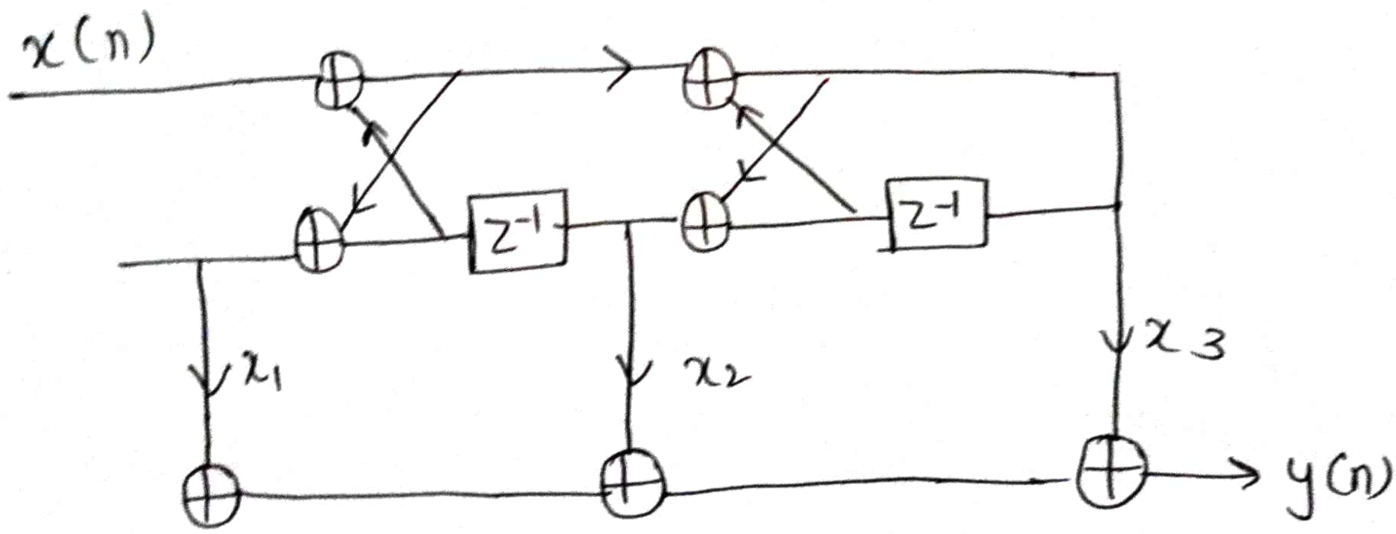


③ Cascade form





③ Lattice form :-



④ Parallel form :-