# Reinforcement Learning

# Reinforcement Learning

- Agent placed in an environment and must learn to behave optimally in it
- Assume that the world behaves like an MDP, except:
  - Agent can act but does not know the transition model
  - Agent observes its current state its reward but doesn't know the reward function
- Goal: learn an optimal policy

# Factors that Make RL Difficult

- Actions have non-deterministic effects
  - which are initially unknown and must be learned
- Rewards / punishments can be infrequent
  - Often at the end of long sequences of actions
  - How do we determine what action(s) were really responsible for reward or punishment? (credit assignment problem)
  - World is large and complex

# Passive vs. Active learning

- Passive learning
  - The agent acts based on a fixed policy $\pi$ and tries to learn how good the policy is by observing the world go by
  - Analogous to policy evaluation in policy iteration
- Active learning
  - The agent attempts to find an optimal (or at least good) policy by exploring different actions in the world
  - Analogous to solving the underlying MDP

# Model-Based vs. Model-Free RL

- *Model based approach to RL:*
  - learn the MDP model (T and R), or an approximation of it
  - use it to find the optimal policy
- *Model free approach to RL:*
  - derive the optimal policy without explicitly learning the model

We will consider both types of approaches

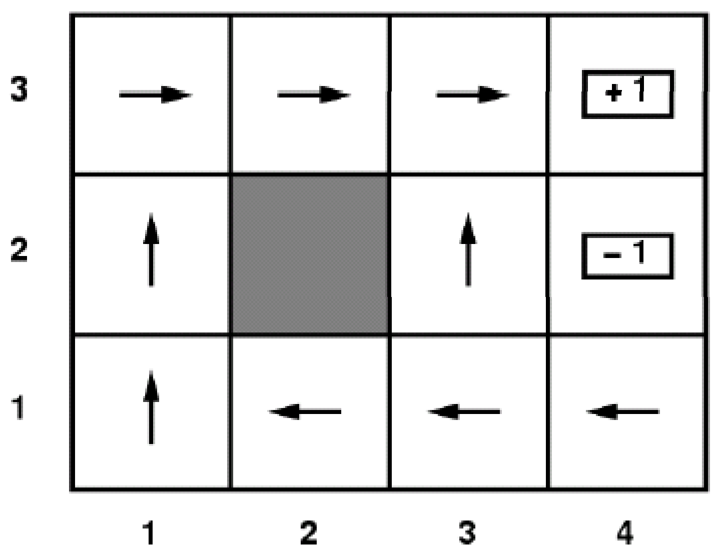
# Passive Reinforcement Learning

- Suppose agent's policy $\pi$ is fixed
- It wants to learn how good that policy is in the world ie. it wants to learn $\mathbf{U^{\pi}(s)}$
- This is just like the policy evaluation part of policy iteration
- The big difference: the agent doesn't know the transition model or the reward function (but it gets to observe the reward in each state it is in)

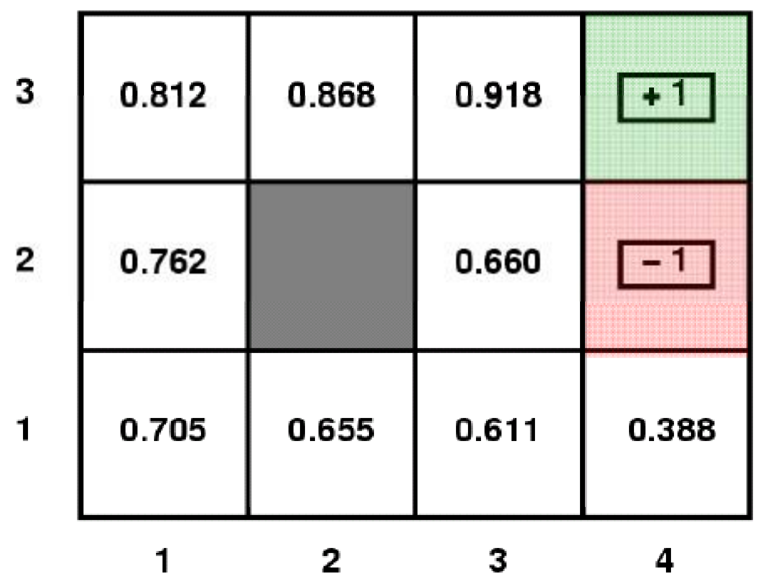
# Passive RL

- Suppose we are given a policy
- Want to determine how good it is

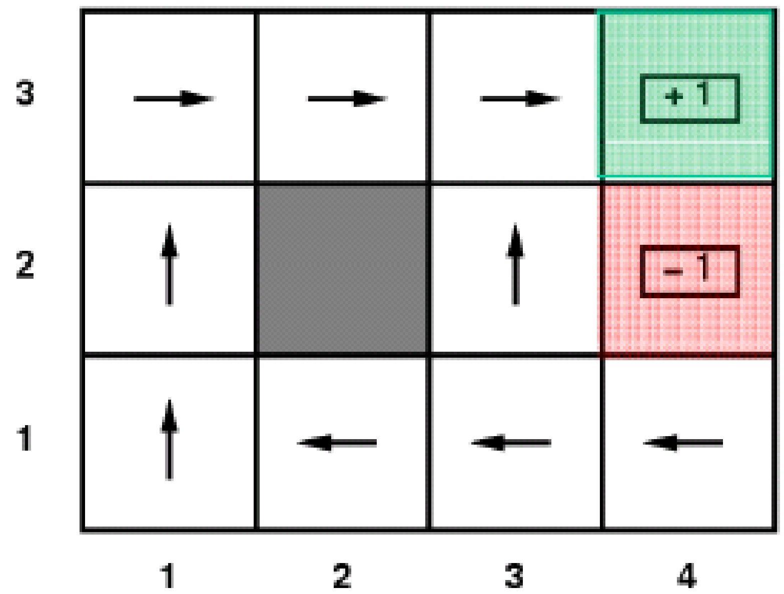
Given $\pi$:




Need to learn $U_\pi(S)$:



# Passive RL




- Given policy π,
  - estimate $U_{\pi}(s)$
- Not given
  - transition matrix, nor
  - reward function!
- Simply follow the policy for many epochs
- Epochs: training sequences

(1,1)→(1,2)→(1,3)→(1,2)→(1,3)→(2,3)→(3,3) →(3,4) +1
(1,1)→(1,2)→(1,3)→(2,3)→(3,3)→(3,2)→(3,3)→(3,4) +1
(1,1)→(2,1)→(3,1)→(3,2)→(4,2) -1

# Appr. 1: Direct Utility Estimation

- Direct utility estimation (model free)
  - Estimate ***$U_\pi(s)$ as average total reward of epochs*** containing s (calculating from s to end of epoch)
- ***Reward to go of a state s***
  - the sum of the (discounted) rewards from that state until a terminal state is reached
- Key: use observed ***reward to go of the state*** as the direct evidence of the actual expected utility of that state

# Direct Utility Estimation

Suppose we observe the following trial:

$(1,1)_{-0.04} \rightarrow (1,2)_{-0.04} \rightarrow (1,3)_{-0.04} \rightarrow (1,2)_{-0.04} \rightarrow (1,3)_{-0.04} \rightarrow (2,3)_{-0.04} \rightarrow (3,3)_{-0.04} \rightarrow (4,3)_{+1}$

The total reward starting at (1,1) is 0.72. We call this a sample of the observed-reward-to-go for (1,1).

For (1,2) there are two samples for the observed-reward-to-go (assuming $\gamma$=1):

1. $(1,2)_{-0.04} \rightarrow (1,3)_{-0.04} \rightarrow (1,2)_{-0.04} \rightarrow (1,3)_{-0.04} \rightarrow (2,3)_{-0.04} \rightarrow (3,3)_{-0.04} \rightarrow (4,3)_{+1}$ [Total: 0.76]
2. $(1,2)_{-0.04} \rightarrow (1,3)_{-0.04} \rightarrow (2,3)_{-0.04} \rightarrow (3,3)_{-0.04} \rightarrow (4,3)_{+1}$ [Total: 0.84]

# Direct Utility Estimation

- Direct Utility Estimation keeps a running average of the observed reward-to-go for each state
- Eg. For state (1,2), it stores (0.76+0.84)/2 = 0.8
- As the number of trials goes to infinity, the sample average converges to the true utility

# Direct Utility Estimation

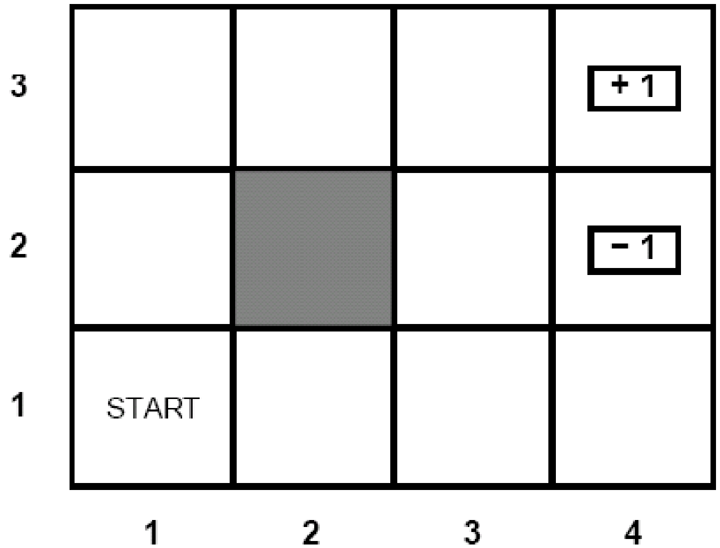
- The big problem with Direct Utility Estimation: it converges very slowly!
- Why?
  - Doesn't exploit the fact that utilities of states are not independent
  - Utilities follow the Bellman equation

$$U_{\pi}(s) = R(s) + \sum_{s'} T(s, \quad (s), s') U_{\pi}(s')$$

Note the dependence on neighboring states

# Direct Utility Estimation

## Using the dependence to your advantage:




Remember that each blank state has R(s) = -0.04

Suppose you know that state (3,3) has a high utility

Suppose you are now at (3,2)

The Bellman equation would be able to tell you that (3,2) is likely to have a high utility because (3,3) is a neighbor.

DEU can’t tell you that until the end of the trial

# Adaptive Dynamic Programming (Model based)

- This method does take advantage of the constraints in the Bellman equation
- Basically learns the transition model **T** and the reward function **R**
- Based on the underlying MDP (**T** and **R**) we can perform policy evaluation (which is part of policy iteration previously taught)

# Adaptive Dynamic Programming

- Recall that policy evaluation in policy iteration involves solving the utility for each state if policy $\pi_i$ is followed.
- This leads to the equations:

$$U_\pi(s) = R(s) + \quad \underset{s'}{\phantom{\sum}} \quad T(s, \quad (s), s')U_\pi(s')$$

- The equations above are linear, so they can be solved with linear algebra in time $O(n^3)$ where n is the number of states

# Adaptive Dynamic Programming

- Make use of policy evaluation to learn the utilities of states
- In order to use the policy evaluation eqn:

$$U_\pi(s) = \underline{R(s)} + \gamma \sum_{s'} \underline{T(s,\pi(s),s')} U_\pi(s')$$

the agent needs to learn the transition model T(s,a,s') and the reward function R(s)

***How do we learn these models?***

# Adaptive Dynamic Programming

- Learning the reward function R(s):

  Easy because it's deterministic. Whenever you see a new state, store the observed reward value as R(s)

- Learning the transition model T(s,a,s'):

  Keep track of how often you get to state s' given that you're in state s and do action a.

  - eg. if you are in s = (1,3) and you execute Right three times and you end up in s'=(2,3) twice, then T(s,Right,s') = 2/3.

# ADP Algorithm

**function** PASSIVE-ADP-AGENT(*percept*) **returns** an action

**inputs**: *percept*, a percept indicating the current state *s′* and reward signal *r′*

**static**: $\pi$, a fixed policy

*mdp*, an MDP with model T, rewards R, discount γ

*U*, a table of utilities, initially empty

$N_{sa}$, a table of frequencies for state-action pairs, initially zero

$N_{sas'}$, a table of frequencies for state-action-state triples, initially zero

*s*, *a* the previous state and action, initially null

**if** *s′* is new **then do** *U[s′]* ← *r′*; *R[s′]* ← *r′*

**if** *s* is not null, **then do**

increment $N_{sa}[s,a]$ and $N_{sas'}[s,a,s']$

for each *t* such that $N_{sas'}[s,a,t]$ is nonzero do

$T[s,a,t] \leftarrow N_{sas'}[s,a,t] \,/\, N_{sa}[s,a]$

*U* ← POLICY-EVALUATION($\pi$, *U*, *mdp*)

**if** TERMINAL?[*s′*] then *s*, *a* ← null **else** *s*, *a* ← *s′*, $\pi[s']$

**return** a

Update reward function

Update transition model

# The Problem with ADP

- Need to solve a system of simultaneous equations – costs $O(n^3)$
  - Very hard to do if you have $10^{50}$ states like in Backgammon
  - Could makes things a little easier with modified policy iteration
- Can we avoid the computational expense of full policy evaluation?

# Temporal Difference Learning

- Instead of calculating the exact utility for a state can we approximate it and possibly make it less computationally expensive?
- Yes we can! Using Temporal Difference (TD) learning
- $U_{\pi}(s) = R(s) + \quad T(s, \quad (s), s')U_{\pi}(s')$
    - $s'$

- Instead of doing this sum over all successors, only adjust the utility of the state based on the successor observed in the trial.
- It does not estimate the transition model – model free

# TD Learning

Example:

- Suppose you see that $U^{\pi}(1,3) = 0.84$ and $U^{\pi}(2,3) = 0.92$ after the first trial.
- If the transition $(1,3) \rightarrow (2,3)$ happens all the time, you would expect to see:

  $U^{\pi}(1,3) = R(1,3) + U^{\pi}(2,3)$

  $\Rightarrow U^{\pi}(1,3) = -0.04 + U^{\pi}(2,3)$

  $\Rightarrow U^{\pi}(1,3) = -0.04 + 0.92 = 0.88$
- Since you observe $U^{\pi}(1,3) = 0.84$ in the first trial, it is a little lower than 0.88, so you might want to "bump" it towards 0.88.

# Aside: Online Mean Estimation

- Suppose that we want to incrementally compute the mean of a sequence of numbers
  - E.g. to estimate the mean of a r.v. from a sequence of samples.

$$\hat{X}_{n+1} = \frac{1}{n+1}\sum_{i=1}^{n+1} x_i = \frac{1}{n}\sum_{i=1}^{n} x_i + \frac{1}{n+1}\left( x_{n+1} - \frac{1}{n}\sum_{i=1}^{n} x_i \right)$$

$$= \hat{X}_n + \frac{1}{n+1}\left(x_{n+1} - \hat{X}_n\right)$$

average of n+1 samples

learning rate

sample n+1

- Given a new sample x(n+1), the new mean is the old estimate (for n samples) plus the weighted difference between the new sample and old estimate

# Temporal Difference Learning (TD)

- TD update for transition from s to s':

$$U_\pi(s) = U_\pi(s) + \alpha(R(s) + \gamma U_\pi(s') - U_\pi(s))$$

learning rate

New (noisy) sample of utility based on next state

- So the update is maintaining a "mean" of the (noisy) utility samples
- If the learning rate decreases with the number of samples (e.g. 1/n) then the utility estimates will eventually converge to true values!

$$U_\pi(s) = R(s) + \gamma \sum_{s'} T(s, a, s') U_\pi(s')$$

# Temporal Difference Update

When we move from state s to s', we apply the following update rule:

$$U_\pi(s) = U_\pi(s) + \alpha(R(s) + \gamma U_\pi(s') - U_\pi(s))$$

This is similar to one step of value iteration

We call this equation a "backup"

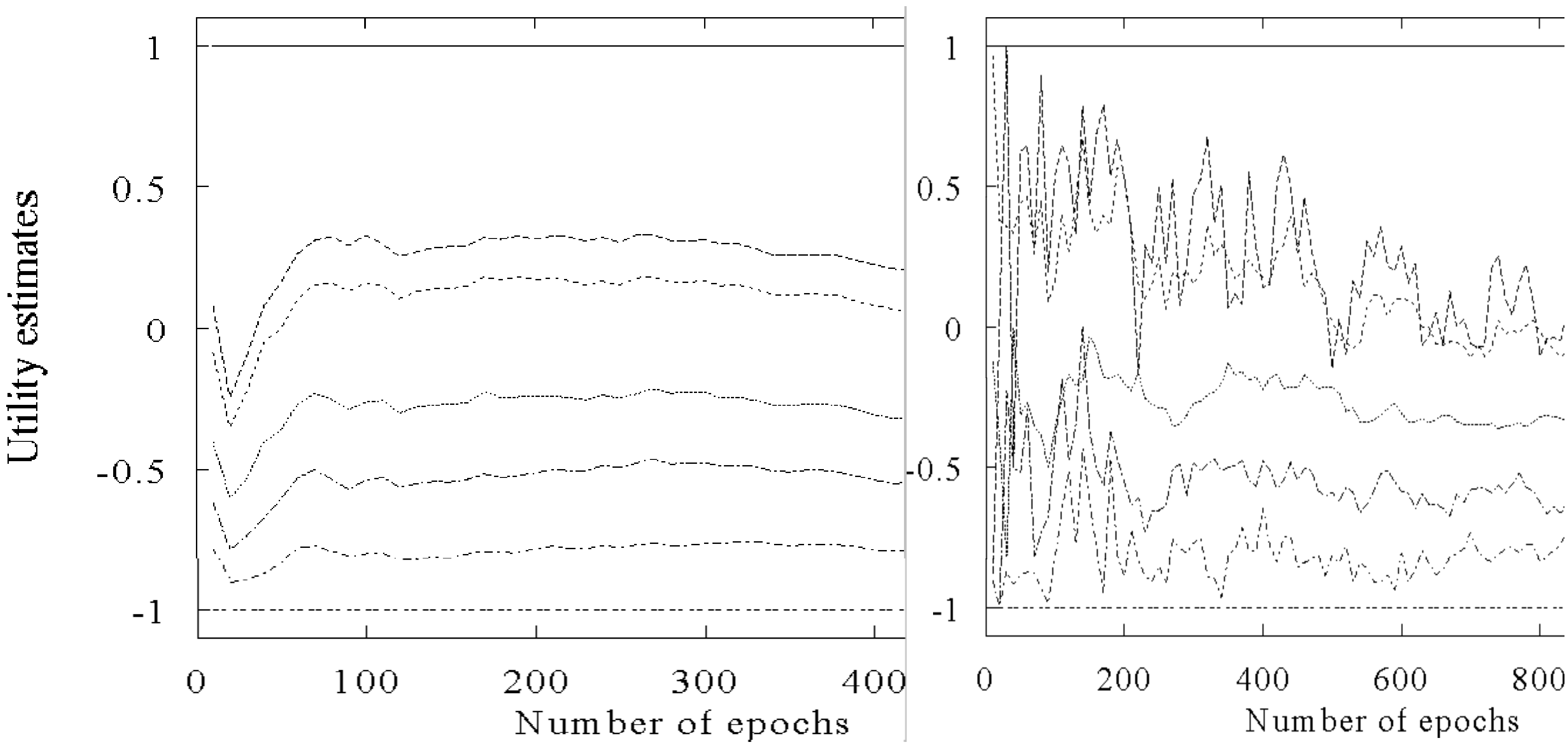
# Convergence

- Since we're using the observed successor s' instead of all the successors, what happens if the transition s $\rightarrow$ s' is very rare and there is a big jump in utilities from s to s'?
- How can $U_{\pi}(s)$ converge to the true equilibrium value?
- Answer: The average value of $U_{\pi}(s)$ will converge to the correct value
- This means we need to observe enough trials that have transitions from s to its successors
- Essentially, the effects of the TD backups will be averaged over a large number of transitions
- Rare transitions will be rare in the set of transitions observed

# Comparison between ADP and TD

- Advantages of ADP:
  - Converges to the true utilities faster
  - Utility estimates don’t vary as much from the true utilities
- Advantages of TD:
  - Simpler, less computation per observation
  - Crude but efficient first approximation to ADP
  - Don’t need to build a transition model in order to perform its updates (this is important because we can interleave computation with exploration rather than having to wait for the whole model to be built first)

# ADP and TD


Utility estimates
1
0.5
0
-0.5
-1
0
100
200
300
400
Number of epochs
1
0.5
0
-0.5
-1
0
200
400
600
800
Number of epochs

# Overall comparisons

- Direct Estimation (model free)
  - Simple to implement
  - Each update is fast
  - Does not exploit Bellman constraints and converges slowly
- Adaptive Dynamic Programming (model based)
  - Harder to implement
  - Each update is a full policy evaluation (expensive)
  - Fully exploits Bellman constraints
  - Fast convergence (in terms of epochs)
- Temporal Difference Learning (model free)
  - Update speed and implementation simliar to direct estimation
  - Partially exploits Bellman constraints---adjusts state to 'agree' with observed successor
    - Not ***all*** possible successors
  - Convergence in between direct estimation and ADP

# What You Should Know

- How reinforcement learning differs from supervised learning and from MDPs
- Pros and cons of:
  - Direct Utility Estimation
  - Adaptive Dynamic Programming
  - Temporal Difference Learning