Unit 3

Probabilistic reasoning in Artificial intelligence

Uncertainty

- Suppose A and B are two statements, If we implement if-then rule to these statements, we might write $A \rightarrow B$, which means if A is true then B is true, or if A is false then B is false, if A is true then B is false, if A is false then B is true.
- But consider a situation where we are not sure about whether A is true or not then we cannot express this statement, this situation is called uncertainty.
- So to represent uncertain knowledge, where we are not sure about the predicates, we need uncertain reasoning or probabilistic reasoning.

Causes of uncertainty

1. Information occurred from unreliable sources.
2. Experimental Errors
3. Equipment fault
4. Temperature variation
5. Climate change.

Probabilistic reasoning

- Probabilistic reasoning is a way of knowledge representation where we apply the concept of probability to indicate the uncertainty in knowledge. In probabilistic reasoning, we combine probability theory with logic to handle the uncertainty.
- In the real world, there are lots of scenarios, where the certainty of something is not confirmed, such as "It will rain today," "behavior of someone for some situations," "A match between two teams or two players." These are probable sentences for which we can assume that it will happen but not sure about it, so here we use probabilistic reasoning.

Need of probabilistic reasoning in A

- When there are unpredictable outcomes.
- When specifications or possibilities of predicates becomes too large to handle.
- When an unknown error occurs during an experiment.

In probabilistic reasoning, there are two ways to solve problems with uncertain knowledge:

- **Bayes' rule**
- **Bayesian Statistics**

Probability

- Probability can be defined as a chance that an uncertain event will occur. It is the numerical measure of the likelihood that an event will occur. The value of probability always remains between 0 and 1 that represent ideal uncertainties.

1. $0 \leq P(A) \leq 1$, where P(A) is the probability of an event A.
2. P(A) = 0, indicates total uncertainty in an event A.
3. P(A) =1, indicates total certainty in an event A.

Probability cont.

We can find the probability of an uncertain event by using the below formula.

$$\text{Probability of occurrence} = \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}}$$

- P(¬A) = probability of a not happening event.
- P(¬A) + P(A) = 1.

Event: Each possible outcome of a variable is called an event.

Sample space: The collection of all possible events is called sample space.

Random variables: Random variables are used to represent the events and objects in the real world.

Prior probability: The prior probability of an event is probability computed before observing new information.

Posterior Probability: The probability that is calculated after all evidence or information has taken into account. It is a combination of prior probability and new information.

Conditional probability

- Conditional probability is a probability of occurring an event when another event has already happened.
- Let's suppose, we want to calculate the event A when event B has already occurred, "the probability of A under the conditions of B", it can be written as:

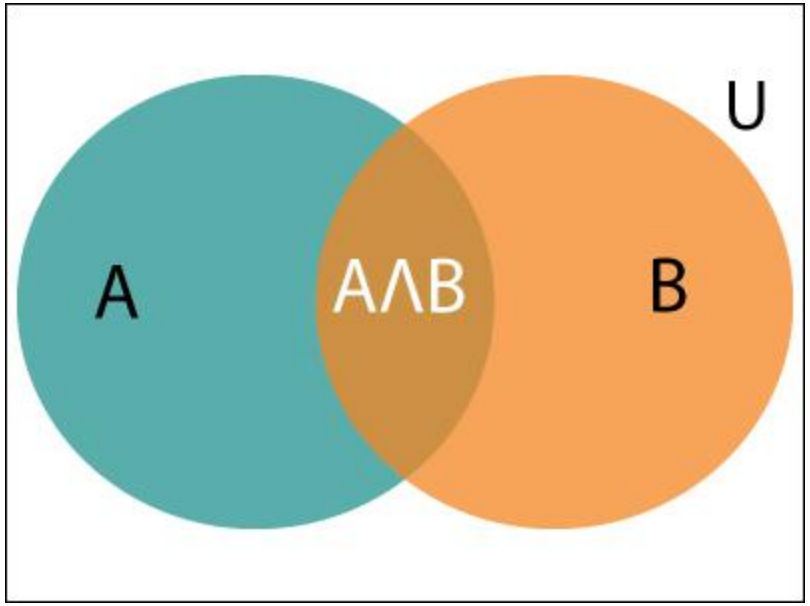
$$P(A|B) = \frac{P(A \wedge B)}{P(B)}$$

Where P($A \wedge B$)= Joint probability of A and B
P(B)= Marginal probability of B.

- If the probability of A is given and we need to find the probability of B, then it will be given as:

$$P(B|A) = \frac{P(A \wedge B)}{P(A)}$$

Venn Diagram



Venn Diagram Example

- **Example:**
- In a class, there are 70% of the students who like English and 40% of the students who likes English and mathematics, and then what is the percent of students those who like English also like mathematics?
- **Solution:**
- Let, A is an event that a student likes Mathematics
- B is an event that a student likes English

$$P(A|B) = \frac{P(A \wedge B)}{P(B)} = \frac{0.4}{0.7} = 57\%$$

- **Hence, 57% are the students who like English also like Mathematics.**

Bayes' theorem

- Bayes' theorem is also known as **Bayes' rule, Bayes' law**, or **Bayesian reasoning**, which determines the probability of an event with uncertain knowledge.
- In probability theory, it relates the conditional probability and marginal probabilities of two random events.
- Bayes' theorem was named after the British mathematician **Thomas Bayes**. The **Bayesian inference** is an application of Bayes' theorem, which is fundamental to Bayesian statistics.
- It is a way to calculate the value of P(B|A) with the knowledge of P(A|B).
- Bayes' theorem allows updating the probability prediction of an event by observing new information of the real world.

Example

- If cancer corresponds to one's age then by using Bayes' theorem, we can determine the probability of cancer more accurately with the help of age.
- Bayes' theorem can be derived using product rule and conditional probability of event A with known event B:
- As from product rule we can write:

$$P(A \wedge B) = P(A|B)\, P(B) \text{ or}$$

- Similarly, the probability of event B with known event A:

$$P(A \wedge B) = P(B|A)\, P(A)$$

- Equating right hand side of both the equations, we will get:

$$P(A|B) = \frac{P(B|A)\, P(A)}{P(B)} \quad \text{....(a)}$$

Example cont.

- The above equation (a) is called as **Bayes' rule** or **Bayes' theorem**. This equation is basic of most modern AI systems for **probabilistic inference**.
- It shows the simple relationship between joint and conditional probabilities. Here,
- P(A|B) is known as **posterior**, which we need to calculate, and it will be read as Probability of hypothesis A when we have occurred an evidence B.
- P(B|A) is called the likelihood, in which we consider that hypothesis is true, then we calculate the probability of evidence.
- P(A) is called the **prior probability**, probability of hypothesis before considering the evidence
- P(B) is called **marginal probability**, pure probability of an evidence.
- In the equation (a), in general, we can write P (B) = P(A)*P(B|Ai), hence the Bayes' rule can be written as

$$P(A_i|B) = \frac{P(A_i) * P(B|A_i)}{\sum_{i=1}^{k} P(A_i) * P(B|A_i)}$$

- Where A1, A2, A3,........, An is a set of mutually exclusive and exhaustive events.

Applying Bayes' rule

- Bayes' rule allows us to compute the single term P(B|A) in terms of P(A|B), P(*B*), and P(A). This is very useful in cases where we have a good probability of these three terms and want to determine the fourth one. Suppose we want to perceive the effect of some unknown cause, and want to compute that cause, then the Bayes' rule becomes:

$$P(cause|effect) = \frac{P(effect|cause)\,P(cause)}{P(effect)}$$

Example-1:

Question: what is the probability that a patient has diseases meningitis with a stiff neck?

Given Data:

- A doctor is aware that disease meningitis causes a patient to have a stiff neck, and it occurs 80% of the time. He is also aware of some more facts, which are given as follows:
- The Known probability that a patient has meningitis disease is 1/30,000.
- The Known probability that a patient has a stiff neck is 2%.

- Let a be the proposition that patient has stiff neck and b be the proposition that patient has meningitis. , so we can calculate the following as:
  - $P(a|b) = 0.8$
  - $P(b) = 1/30000$
  - $P(a) = .02$

$$P(b|a) = \frac{P(a|b)P(b)}{P(a)} = \frac{0.8*(\frac{1}{30000})}{0.02} = 0.001333333.$$

- Hence, we can assume that 1 patient out of 750 patients has meningitis disease with a stiff neck.

Example-2:

Question: From a standard deck of playing cards, a single card is drawn. The probability that the card is king is 4/52, then calculate posterior probability P(King|Face), which means the drawn face card is a king card.

Solution:

$$P(king|face) = \frac{P(Face|king) * P(King)}{P(Face)} \quad ....(i)$$

- P(king): probability that the card is King= 4/52= 1/13
- P(face): probability that a card is a face card= 3/13
- P(Face|King): probability of face card when we assume it is a king = 1
- Putting all values in equation (i) we will get:

$$P(king|face) = \frac{1 * (\frac{1}{13})}{(\frac{3}{13})}$$ **= 1/3, it is a probability that a face card is a king card.**

Application of Bayes' theorem in Artifici intelligence

Following are some applications of Bayes' theorem:

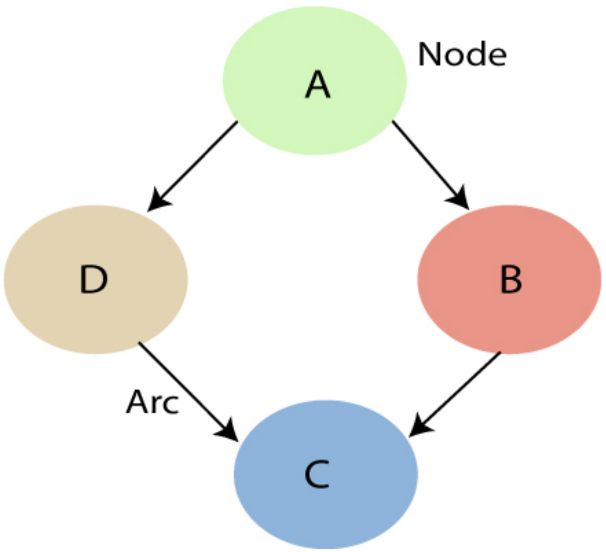
- It is used to calculate the next step of the robot when the already executed step is given.
- Bayes' theorem is helpful in weather forecasting.
- It can solve the Monty Hall problem.

Bayesian Belief Network in artificial intellig

- Bayesian belief network is key computer technology for dealing with probabilistic events and to solve a problem which has uncertainty. We can define a Bayesian network as:
- "A Bayesian network is a probabilistic graphical model which represents a set of variables and their conditional dependencies using a **directed acyclic graph**."
- It is also called a **Bayes network, belief network, decision network**, or **Bayesian model**.
- Bayesian networks are probabilistic, because these networks are built from a **probability distribution**, and also use probability theory for prediction and anomaly detection.
- It can also be used in various tasks including **prediction, anomaly detection, diagnostics, automated insight, reasoning, time series prediction**, and **decision making under uncertainty**.

Bayesian Network cont.

- Bayesian Network can be used for building models from data and experts opinions, and it consists of two parts:
- **Directed Acyclic Graph**
- **Table of conditional probabilities.**
- The generalized form of Bayesian network that represents and solve decision problems under uncertain knowledge is known as an **Influence diagram**.
- **A Bayesian network graph is made up of nodes and Arcs (directed links), where:**



Bayesian Network cont.

- Each **node** corresponds to the random variables, and a variable can be **continuous** or **discrete**.
- **Arc or directed arrows** represent the causal relationship or **conditional probabilities** between random variables. These directed links or arrows connect the pair of nodes in the graph.

  These links represent that one node directly influence the other node, and if there is no directed link that means that nodes are independent with each other

  - **In the above diagram, A, B, C, and D are random variables represented by the nodes of the network graph.**
  - **If we are considering node B, which is connected with node A by a directed arrow, then node A is called the parent of Node B.**
  - **Node C is independent of node A.**

Bayesian Network Components

- The Bayesian network has mainly two components:
- **Causal Component**
- **Actual numbers**
- Each node in the Bayesian network has condition probability distribution $\mathbf{P(X_i \mid Parent(X_i))}$, which determines the effect of the parent on that node.
- Bayesian network is based on Joint probability distribution and conditional probability. So let's first understand the joint probability distribution:

Joint probability distribution

- If we have variables x1, x2, x3,....., xn, then the probabilities of a different combination of x1, x2, x3.. xn, are known as Joint probability distribution.

$$\mathbf{P[x_1, x_2, x_3, \ldots\ldots, x_n]},$$

it can be written as the following way in terms of the joint probability distribution.

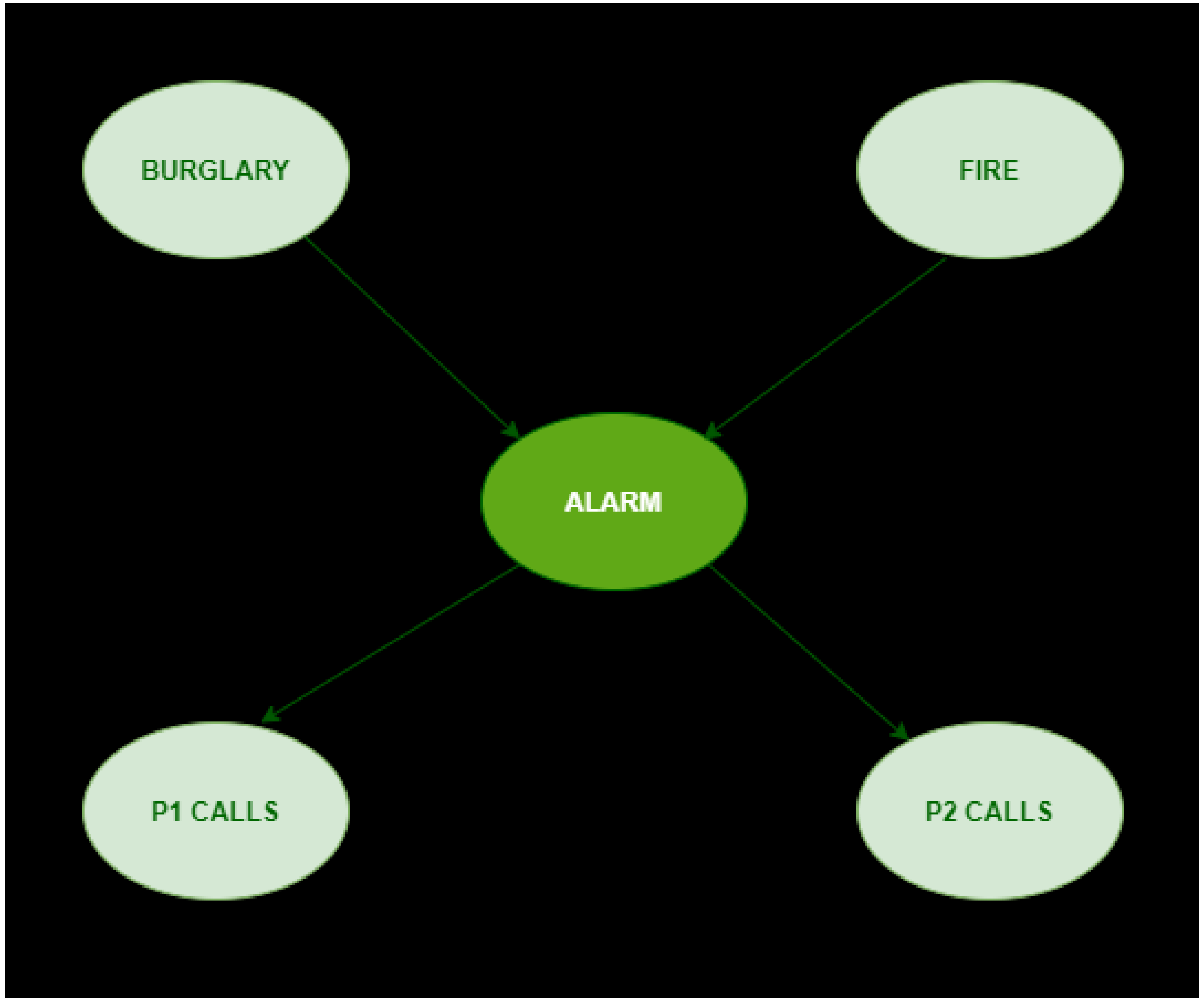
$$\mathbf{= P[x_1| x_2, x_3, \ldots\ldots, x_n]P[x_2, x_3, \ldots\ldots, x_n]}$$

$$\mathbf{= P[x_1| x_2, x_3, \ldots\ldots, x_n]P[x_2|x_3, \ldots\ldots, x_n] \ldots\ldots P[x_{n-1}|x_n]P[x_n].}$$

- In general for each variable Xi, we can write the equation as:

$$\mathbf{P(X_i|X_{i-1}, \ldots\ldots\ldots, X_1) = P(X_i\,|Parents(X_i\,))}$$

Example



- In the above figure, we have an alarm ‘A’ – a node, say installed in a h person ‘gfg’, which rings upon two probabilities i.e burglary ‘B’ and fire ‘F’, which are – parent nodes of the alarm node. The alarm is the parent node of two probabilities P1 calls ‘P1’ & P2 calls ‘P2’ person nodes.
- Upon the instance of burglary and fire, ‘P1’ and ‘P2’ call person ‘gfg’, respectively. But, there are few drawbacks in this case, as sometimes ‘P1’ may forget to call the person ‘gfg’, even after hearing the alarm, as he has a tendency to forget things, quick. Similarly, ‘P2’, sometimes fails to call the person ‘gfg’, as he is only able to hear the alarm, from a certain distance.

- **Q)** Find the probability that ‘P1’ is true (P1 has called ‘gfg’), ‘P2’ is true (P2 has called ‘gfg’) when the alarm ‘A’ rang, but no burglary ‘B’ and fire ‘F’ has occurred.
- => **P (P1, P2, A, ~B, ~F)** [where- P1, P2 & A are ‘true’ events and ‘~B’ & ‘~F’ are ‘false’ events]
- [**Note:** The values mentioned below are neither calculated nor computed. They have observed values]
- ***Burglary ‘B’ –***
- **P (B=T) = 0.001** (‘B’ is true i.e burglary has occurred)
- **P (B=F) = 0.999** (‘B’ is false i.e burglary has not occurred)
- ***Fire ‘F’ –***
- **P (F=T) = 0.002** (‘F’ is true i.e fire has occurred)
- **P (F=F) = 0.998** (‘F’ is false i.e fire has not occurred)

Alarm ‘A’ –

B	**F**	**P (A=T)**	**P (A=F)**
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
F	F	**0.001**	0.999

•The alarm ‘A’ node can be ‘true’ or ‘false’ (i.e may have rung or may not have rung). It has two parent nodes burglary ‘B’ and fire ‘F’ which can be ‘true’ or ‘false’ (i.e may have occurred or may not have occurred) depending upon different conditions.

Person ‘P1’ –

A	P (P1=T)	P (P1=F)
T	**0.95**	0.05
F	0.05	0.95

- The person ‘P1’ node can be ‘true’ or ‘false’ (i.e may have called the person ‘gfg’ or not) . It has a parent node, the alarm ‘A’, which can be ‘true’ or ‘false’ (i.e may have rung or may not have rung ,upon burglary ‘B’ or fire ‘F’).

Person ‘P2’ –

A	**P (P2=T)**	**P (P2=F)**
T	**<u>0.80</u>**	0.20
F	0.01	0.99

- The person ‘P2’ node can be ‘true’ or false’ (i.e may have called the person ‘gfg’ or not). It has a parent node, the alarm ‘A’, which can be ‘true’ or ‘false’ (i.e may have rung or may not have rung, upon burglary ‘B’ or fire ‘F’).

- **Solution:** Considering the observed probabilistic scan –
- With respect to the question — **P (P1, P2, A, ~B, ~F)** , we need to get the probability of ‘P1’. We find it with regard to its parent node – alarm ‘A’. To get the probability of ‘P2’, we find it with regard to its parent node — alarm ‘A’.
- We find the probability of alarm ‘A’ node with regard to ‘~B’ & ‘~F’ since burglary ‘B’ and fire ‘F’ are parent nodes of alarm ‘A’.
- From the observed probabilistic scan, we can deduce –
- **P (P1, P2, A, ~B, ~F)**
- **= P (P1/A) * P (P2/A) * P (A/~B~F) * P (~B) * P (~F)**
- **= 0.95 * 0.80 * 0.001 * 0.999 * 0.998**
- **= 0.00075**
-